

ΕΝΟΤΗΤΑ 2: Παραγοντοποίηση-Ρητές Αλγεβρικές Παραστάσεις

Οδηγίες: Από το σχολικό βιβλίο (1^ο τεύχος) να μελετήσετε τη θεωρία και τις ασκήσεις σελ. 43-89. Ακολουθούν ενδεικτικές επαναληπτικές ασκήσεις. Δοκιμάστε να τις λύσετε αφού μελετήσετε ξανά τη θεωρία του κεφαλαίου.

ΕΠΑΝΑΛΗΠΤΙΚΕΣ ΑΣΚΗΣΕΙΣ

1. Να αναλύσετε πλήρως σε γινόμενο πρώτων παραγόντων τα πολυώνυμα:

<p>1) $4\chi + 4\psi + 8\omega = 4(\chi + \psi + 2\omega)$</p> <p>3) $9\chi^2 - 16\psi^2 = (3\chi)^2 - (4\psi)^2 = (3\chi - 4\psi)(3\chi + 4\psi)$</p> <p>5) $25\chi^2 + 40\chi + 16 = (5\chi + 4)^2 = (5\chi)^2 + 2 \cdot 5\chi \cdot 4 + 4^2$</p> <p>7) $2\chi^2 - 14\chi + 20 = 2(\chi^2 - 7\chi + 10) = 2(\chi - 5)(\chi - 2)$</p>	<p>2) $3\chi - 3\psi - \omega\chi + \omega\psi = 3(\chi - \psi) - \omega(\chi - \psi) = (\chi - \psi)(3 - \omega)$</p> <p>4) $\chi^2 - \chi - 30 = (\chi - 6)(\chi + 5)$</p> <p>6) $\psi^3 - 25\psi = \psi(\psi^2 - 25) = \psi(\psi - 5)(\psi + 5)$</p> <p>8) $-\psi^2 + 8\psi - 15 = -(\psi^2 - 8\psi + 15) = -(\psi - 5)(\psi - 3)$</p>
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2. Να αναλύσετε πλήρως σε γινόμενο πρώτων παραγόντων τα πολυώνυμα:

<p>1) $\alpha(\alpha - 2) - \beta(\beta - 2) = \alpha^2 - 2\alpha - \beta^2 + 2\beta = (\alpha - \beta)(\alpha + \beta) - 2(\alpha - \beta) = (\alpha - \beta)(\alpha + \beta - 2)$</p> <p>3) $(\chi - 3\omega)^2 + (\chi - 3\omega) - 6 = (\chi - 3\omega)^2 + 3(\chi - 3\omega) - 2(\chi - 3\omega) = (\chi - 3\omega + 3)(\chi - 3\omega - 2)$</p> <p>5) $16\chi^4 - 81\psi^4 = (4\chi^2)^2 - (9\psi^2)^2 = (4\chi^2 - 9\psi^2)(4\chi^2 + 9\psi^2) = (2\chi - 3\psi)(2\chi + 3\psi)(4\chi^2 + 9\psi^2)$</p>	<p>2) $\chi^2 - 6\chi + 9 - 2\beta\chi + 6\beta = (\chi - 3)^2 - 2\beta(\chi - 3) = (\chi - 3)(\chi - 3 - 2\beta)$</p> <p>4) $\chi^2 - 6\psi - 1 - 9\psi^2 + 4 - 4\chi = (\chi^2 - 4\chi + 4) - (9\psi^2 + 6\psi + 1) = (\chi - 2)^2 - (3\psi + 1)^2 = (\chi - 2 - 3\psi - 1)(\chi - 2 + 3\psi + 1) = (\chi - 3\psi - 3)(\chi + 3\psi - 1)$</p> <p>6) $3\rho^2 - 3\omega^2 - \omega^2 - 2\rho\omega - \rho^2 = 3(\rho^2 - \omega^2) - (\omega^2 + 2\rho\omega + \rho^2) = 3(\rho - \omega)(\rho + \omega) - (\omega + \rho)^2 = (\rho + \omega)[3(\rho - \omega) - (\omega + \rho)] = (\rho + \omega)(3\rho - 3\omega - \omega - \rho) = (\rho + \omega)(2\rho - 4\omega) = 2(\rho + \omega)(\rho - 2\omega)$</p>
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$$7) a^2(a-5) + (a-5)(3a-2) - 25 + a^2 =$$

$$a^2(a-5) + (a-5)(3a-2) - (a^2 - 25) =$$

$$a^2(a-5) + (a-5)(3a-2) - (a-5)(a+5) =$$

$$(a-5)(a^2 + 3a - 2 - a - 5) = (a-5)(a^2 + 2a - 7)$$

$$9) (x^2 - 6x + 3)^2 - (x-9)^2 =$$

$$(x^2 - 6x + 3 - x + 9)(x^2 - 6x + 3 + x - 9) =$$

$$(x^2 - 7x + 12)(x^2 - 5x - 6) = (x-4)(x-3)(x-6)(x+1)$$

$$8) x^2 - 6x\psi + 9\psi^2 - 4\omega^2 = (x-3\psi)^2 - (2\omega)^2$$

$$= (x-3\psi-2\omega)(x-3\psi+2\omega)$$

$$10) 4(x-1) + 9x^2(1-x) = 4(x-1) - 9x^2(x-1)$$

$$= (4-9x^2)(x-1)$$

$$= (2-3x)(2+3x)(x-1)$$

3. Χρησιμοποιώντας πλήρη παραγοντοποίησης γινόμενο ή με άλλο τρόπο να βρείτε τη τιμή του πολυωνύμου $A = 2x^3 + 6x\psi^2 - 2\psi^3 - 6x^2\psi$ για $x=101$ και $\psi=99$.

$$A = 2(x^3 - \psi^3) + 6x\psi(\psi - x)$$

$$A = 2(x-\psi)(x^2 + x\psi + \psi^2) - 6x\psi(x-\psi)$$

$$A = (x-\psi)(2x^2 + 2x\psi + 2\psi^2 - 6x\psi)$$

$$A = (x-\psi)(2x^2 - 4x\psi + 2\psi^2) = 2(x-\psi)(x^2 - 2x\psi + \psi^2) = 2(x-\psi)(x-\psi)^2$$

$$= 2(x-\psi)^3$$

$$x=101 \text{ και } \psi=99$$

$$A = 2(101-99)^3$$

$$A = 2 \cdot 2^3 = 2^4 = 16$$

4. Να λύσετε τις εξισώσεις:

$$1) x^2 - 8x = 0$$

$$x(x-8) = 0$$

$$\begin{cases} x=0 \\ x-8=0 \\ x=8 \end{cases}$$

$$2) x^2 - 64 = 0$$

$$(x-8)(x+8) = 0$$

$$\begin{cases} x-8=0 \Rightarrow x=8 \\ x+8=0 \Rightarrow x=-8 \end{cases}$$

$$3) x^2 - 2x = 15$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$\begin{cases} x-5=0 \\ x=5 \\ x+3=0 \\ x=-3 \end{cases}$$

$$4) (x+5)(x^2 - 2x - 3)(2x-5) = 0$$

$$(x+5)(x-3)(x+1)(2x-5) = 0$$

$$\begin{cases} x+5=0 \\ x=-5 \\ x-3=0 \\ x=3 \\ x+1=0 \\ x=-1 \\ 2x-5=0 \\ 2x=5 \\ x=\frac{5}{2} \end{cases}$$

$$5) 25\psi^2 - 20\psi + 4 = 0$$

$$(5\psi - 2)^2 = 0$$

$$5\psi - 2 = 0 \Rightarrow 5\psi = 2$$

$$\Rightarrow \psi = \frac{2}{5} \text{ διπλή ρίζα}$$

$$6) 2x^2 - x - 10 = 0 \quad a=2 \quad b=-1 \quad \gamma=-10$$

$$\Delta = b^2 - 4 \cdot a \cdot \gamma = (-1)^2 - 4 \cdot 2 \cdot (-10)$$

$$\Delta = 1 + 80 = 81$$

$$x_1, x_2 = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a} = \frac{+1 \pm 9}{4}$$

$$x_1 = \frac{1+9}{4} = \frac{10}{4} = \frac{5}{2}$$

$$x_2 = \frac{1-9}{4} = \frac{-8}{4} = -2$$

$$7) 3(3x-4) = x(x+2)$$

$$9x-12 = x^2+2x$$

$$0 = x^2+2x-9x+12$$

$$0 = x^2-7x+12$$

$$0 = (x-4)(x-3)$$

$$x=4 \quad x=3$$

$$8) 3x^2 - 5x + 4 = 0 \quad a=3 \quad b=-5 \quad \gamma=4$$

$$\Delta = b^2 - 4 \cdot a \cdot \gamma = (-5)^2 - 4 \cdot 3 \cdot 4 = 25 - 48$$

$$\Delta = -23 < 0$$

Η εξίσωση δεν έχει ρίζες στο σύνολο των πραγματικών αριθμών.

5. Να απλοποιήσετε τα κλάσματα:

$$1) \frac{x^2 - 25}{2x - 10}$$

$$= \frac{(x-5)(x+5)}{2(x-5)}$$

$$= \frac{x+5}{2}$$

$x \neq 5$
ή $x \in \mathbb{R} - \{5\}$

$$2) \frac{5a^2b - 5ab^2}{a^3b - ab^3} = \frac{5ab(a-b)}{ab(a^2 - b^2)}$$

$$= \frac{5ab(a-b)}{ab(a-b)(a+b)}$$

$$= \frac{5}{a+b}$$

$a \neq 0$
 $b \neq 0$
 $a \neq b$
 $a \neq -b$

6. Να κάνετε τις πράξεις:

$$1) \frac{x\psi^2}{x^2 + 3x - 18} \cdot \frac{4x + 24}{x\psi}$$

$$= \frac{\cancel{x}\psi^2}{(x+6)(x-3)} \cdot \frac{4\cancel{x}(x+6)}{\cancel{x}\psi}$$

$$= \frac{4\psi}{x-3}$$

$x \neq 0$
 $\psi \neq 0$
 $x \neq -6$
 $x \neq 3$

$$2) \frac{x^2 - 8x + 12}{x^2 - 36} : \frac{3x - 6}{x^2 + 5x - 6} =$$

$$\frac{(x-6)(x-2)}{(x-6)(x+6)} \cdot \frac{3(x-2)}{(x+6)(x-1)} =$$

$$\frac{\cancel{(x-6)}(x-2)}{\cancel{(x-6)}(x+6)} \cdot \frac{\cancel{3}(x-2)}{3(x-1)}$$

$$= \frac{x-1}{3}$$

$x \neq 6$
 $x \neq -6$
 $x \neq 1$
 $x \neq 2$

$$3) \frac{2x}{x^2-25} + \frac{1}{5-x} - \frac{3}{x^2+5x}$$

$$\frac{\frac{x}{2x}}{(x-5)(x+5)} - \frac{1}{x-5} - \frac{3}{x(x+5)} =$$

$$\frac{2x^2 - x^2 - 5x - 3x + 15}{x(x-5)(x+5)} =$$

$$\frac{x^2 - 8x + 15}{x(x-5)(x+5)} =$$

$$\frac{(x-3)(x-5)}{x(x-5)(x+5)} =$$

$$\frac{x-3}{x(x+5)}$$

$$4) \frac{3x^2-3}{x^3+x^2-2x} : \left(\frac{3}{x^2-4} + \frac{1}{x+2} \right)$$

$$\in \kappa \pi \neq 0$$

$$x \neq 0$$

$$x \neq 5$$

$$x \neq -5$$

$$\frac{3(x^2-1)}{x(x^2+x-2)} : \left(\frac{1}{3} + \frac{x-2}{x+2} \right) =$$

$$\frac{3(x-1)(x+1)}{x(x+2)(x-1)} : \frac{3+x-2}{(x-2)(x+2)} =$$

$$\frac{3(x-1)(x+1)}{x(x+2)(x-1)} : \frac{x+1}{(x-2)(x+2)} = \left. \begin{array}{l} x \neq 0 \\ x \neq -2 \\ x \neq 1 \\ x \neq -2 \end{array} \right\}$$

$$\frac{3(x-1)(x+1)}{x(x+2)(x-1)} \cdot \frac{(x-2)(x+2)}{x+1} = \left. \begin{array}{l} x \neq -1 \end{array} \right\}$$

$$\frac{3(x-2)}{x} = \frac{3x-6}{x}$$

7. Να απλοποιήσετε τις πιο κάτω παραστάσεις:

$$1) \frac{\frac{x-9\psi}{\psi} \cdot \frac{\psi}{x}}{x^2-6x\psi+9\psi^2} =$$

$$\frac{x^2-9\psi^2}{x\psi} =$$

$$\frac{x^2-6x\psi+9\psi^2}{\psi^2}$$

$$\frac{(x-3\psi)(x+3\psi)}{x\psi} =$$

$$\frac{(x-3\psi)^2}{\psi^2} \quad \left. \begin{array}{l} x \neq 0 \\ \psi \neq 0 \\ x \neq 3\psi \end{array} \right\}$$

$$\frac{(x-3\psi)(x+3\psi)\psi^2}{x\psi(x-3\psi)^2} =$$

$$\frac{(x+3\psi)\psi}{x(x-3\psi)}$$

$$2) \frac{x^2-16}{x^2+3x-4} = \frac{(x-4)(x+4)}{(x+4)(x-1)}$$

$$\frac{x^2-4x}{x^2} = \frac{x(x-4)}{x^2}$$

$$= \frac{(x-4)(x+4)x^2}{(x+4)(x-1)x(x-4)} \quad \left. \begin{array}{l} x \neq -4 \\ x \neq 1 \\ x \neq 0 \end{array} \right\}$$

$$= \frac{x}{x-1} \quad x \neq 4$$

8. Να λύσετε τις εξισώσεις:

$$1) \frac{x-2}{x} + \frac{4}{x-2} = \frac{8}{x^2-2x}$$

$$\frac{x-2}{x} + \frac{4}{x-2} = \frac{8}{x(x-2)}$$

$$\frac{x^2-2x-2x+4+4x}{x(x-2)} = \frac{8}{x(x-2)} \quad \left| \begin{array}{l} x \neq 0 \\ x \neq 2 \end{array} \right.$$

$$x^2+4=8$$

$$x^2-4=0$$

$$(x-2)(x+2)=0$$

$$x=2 \text{ απο. } \boxed{x=-2} \text{ δευτενη}$$

$$3) \frac{y-5}{y+5} - \frac{y}{y-5} = \frac{y^2+25}{25-y^2}$$

$$\frac{y-5}{y+5} - \frac{y}{y-5} = \frac{-y^2-25}{(y-5)(y+5)}$$

$$\frac{3y-15-y^2-5y}{(y-5)(y+5)} = \frac{-y^2-25}{(y-5)(y+5)}$$

$$-y^2-2y-15 = -y^2-25$$

$$-2y = -10$$

$$y = 5 \text{ απο.}$$

Αδυνατη δεν εκκ. जुग्न.

$$5) \frac{p}{p-1} + \frac{6}{p^2-1} = 4$$

$$\frac{p+1}{p-1} + \frac{1}{p+1} = \frac{(p-1)(p+1)}{1}$$

$$\frac{p^2+p+6}{(p-1)(p+1)} = \frac{4(p^2-1)}{(p-1)(p+1)}$$

$$p^2+p+6 = 4p^2-4$$

$$0 = 3p^2-p-10$$

$$a=3 \quad b=-1 \quad \gamma=-10$$

$$\Delta = b^2-4a\gamma = (-1)^2-4 \cdot 3 \cdot (-10)$$

$$\Delta = 1+120 = 121$$

$$p_1, p_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{+1 \pm 11}{6}$$

$$p_1 = \frac{1+11}{6} = 2$$

$$p_2 = \frac{1-11}{6} = -\frac{10}{6} = -\frac{5}{3}$$

$$2) \frac{y+2}{y} = \frac{y+3}{y+4} - \frac{4}{y^2+4y}$$

$$\frac{y+2}{y} = \frac{y+3}{y+4} - \frac{4}{y(y+4)} \quad \left| \begin{array}{l} y \neq 0 \\ y \neq -4 \end{array} \right.$$

$$\frac{y^2+4y+2y+8}{y(y+4)} = \frac{y^2+3y-4}{y(y+4)}$$

$$y^2+6y+8 = y^2+3y-4$$

$$3y = -12$$

$$y = -4 \text{ απο. ΑΔΥΝΑΤΗ}$$

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$$4) \frac{2y}{y^2+y} = 1 - \frac{2}{y+1}$$

$$\frac{2y}{y(y+1)} = 1 - \frac{2}{y+1}$$

$$\frac{2y}{y(y+1)} = \frac{y^2+y-2y}{y(y+1)}$$

$$2y = y^2 - y$$

$$0 = y^2 - 3y$$

$$0 = y(y-3)$$

$$y=0 \quad \boxed{y=3} \text{ δευτενη}$$

απο.

$$6) \frac{3}{\omega^2-3\omega-4} = \frac{2\omega+5}{\omega^2+2\omega^2+\omega} + \frac{4}{\omega^2-4\omega}$$

$$\frac{3}{(\omega-4)(\omega+1)} = \frac{2\omega+5}{\omega(\omega^2+2\omega+1)} + \frac{4}{\omega(\omega-4)}$$

$$\frac{3}{(\omega-4)(\omega+1)} = \frac{2\omega+5}{\omega(\omega+1)^2} + \frac{4}{\omega(\omega-4)}$$

$$\frac{3\omega^2+3\omega}{\omega(\omega-4)(\omega+1)^2} = \frac{-2\omega^2-8\omega+5\omega-20+4\omega^2+8\omega+4}{\omega(\omega-4)(\omega+1)^2}$$

$$3\omega^2+3\omega = 6\omega^2+5\omega-16$$

$$0 = 3\omega^2+2\omega-16$$

$$a=3 \quad b=2 \quad \gamma=-16$$

$$\Delta = 2^2-4 \cdot 3 \cdot (-16) = 4+192 = 196$$

$$\omega_1, \omega_2 = \frac{-2 \pm 14}{6}$$

$$\omega_1 = \frac{-2-14}{6} = -\frac{16}{6} = -\frac{8}{3}$$

$$\omega_2 = \frac{-2+14}{6} = \frac{12}{6} = 2$$

$$7) \left(\frac{x+1}{x-1}\right)^2 - 4\frac{x+1}{x-1} + 3 = 0$$

$$\frac{x^2 + 2 \cdot x \cdot 1 + 1^2}{(x-1)^2} - \frac{\overbrace{x-1}^{x-1} \cdot \overbrace{4x+4}^{(x-1)^2}}{x-1} + \frac{3}{1} = 0$$

$$\frac{x^2 + 2x + 1 - (x-1)(4x+4) + 3(x-1)^2}{(x-1)^2} = 0$$

$$\boxed{x \neq 1}$$

$$x^2 + 2x + 1 - (4x^2 + 4x - 4x - 4) + 3(x^2 - 2 \cdot x \cdot 1 + 1^2) = 0$$

$$x^2 + 2x + 1 - 4x^2 + 4 + 3x^2 - 6x + 3 = 0$$

$$-4x + 8 = 0$$

$$-4x = -8$$

$$\boxed{x=2} \text{ δεικν}$$

$$8) \frac{3}{k+2} = \frac{2}{k} + \frac{k-4}{k^2+2k}$$

$$\frac{\overbrace{3}^k}{k+2} = \frac{\overbrace{2}^{k+2}}{k} + \frac{\overbrace{k-4}^1}{k(k+2)}$$

$$k \neq 0 \\ k \neq -2$$

$$\frac{3k}{k(k+2)} = \frac{2k+k-k}{k(k+2)}$$

$$3k = 3k$$

$$\boxed{0k=0}$$

ΑΟΡΙΣΤΗ ΕΞΙΣΩΣΗ

ΑΠΕΙΡΕΣ ΠΥΞΕΙΣ